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On strong blowing into an incompressible airstream

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An experimental study of distributed air-injection from a porous section of a flat plate into a uniform incompressible airflow is described. The relative mass flow rates of the injection varied between 0·008 and 0·053 (strong injection) and the blowing was fairly uniformly distributed. In the resulting flow field, which was predominantly laminar except near the dividing streamline, where unsteadiness prevailed, velocity profile and pressure measurements were taken and the position of the dividing streamline thereby estimated. Overall the results agree fairly well with the steady laminar theory for strong normal blowing, outlined in §2, although for the strongest blow some signs of separation some way upstream of the blow are apparent.

1. Introduction

In recent years much interest has been shown in the effects that injection of gas through a porous section of a body surface can have on the high Reynolds number flow over the surface. The influences on boundary-layer attachment, heat transfer to the surface and aerodynamic load on the body depend critically of course on the mass flow rate through, and dimensions of, the porous section and many of the possible problems in this field have been examined by previous authors. For example, the practical aspects of strong distributed injection into turbulent boundary layers have been investigated by McQuaid (1967), Mugalev (1959*a*) and Simpson, Kays & Moffat for incompressible flow and by Fernandez & Zukoski (1969) and Mugalev (1959*b*) in supersonic conditions. Also in compressible flow Bott (1968) and Hartunian & Spencer (1966, 1967) have reported on massive distributed blowing experiments, while Goldstein (1971) has reviewed the field of slot-injection for both supersonic and subsonic turbulent flows and for a very wide range of blowing conditions. In this paper 'strong' injection is defined mathematically by $1 \gg V_w/U_\infty \gg Re^{-\frac{1}{2}}$ whereas 'massive' refers to injection rates V_w/U_∞ of order unity, where V_w is the blowing speed, U_∞ is the mainstream speed and Re ($\gg 1$) the Reynolds number based on a characteristic length L. On the theoretical side comprehensive lists of references on the various methods used may be found in Smith & Stewartson (1973) and Inger & Gaitatzes (1971).

The experiments to be described herein were concerned with strong distributed air-to-air injection, for values of $\delta = V_w/U_\infty$ between 0·008 and 0·053 and with

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$Re \approx 2\times 10^4$. In common with many other researchers we concentrated our attention on flow past a flat plate in order to study more directly the effects attributable to the blowing, and attempts were made to ensure uniformity in the distribution of blow and two-dimensionality in the flow field. However, in contrast with the previous measurements reported in the literature, the experiments were conducted in a low-speed wind tunnel and with laminar conditions established as far as possible in the mainstream, leading-edge boundary layer and air injection supply (see figure 1). The techniques employed to study the flow field produced included hot-wire traverses of the wall layers to determine velocity profiles at various stations within and ahead of the blown region and measurements of the pressure distribution at these stations, as described in § 3.

Our principal objective in this investigation was to compare both qualitatively and quantitatively the observed flow characteristics with the predictions of the theoretical model for steady laminar subsonic flows developed by the author (1972) and by Wallace & Kemp (1969), following the work of Cole & Aroesty (1968), and thus to gauge the relevance and limitations of the theory. For convenience, a short note on the theory is presented in § 2. As we shall see, because of the unsteadiness present in the blown-off boundary layer in practice and also the asymptotic nature of the theory, the intended comparisons must at first be regarded as tentative. The agreement in terms of velocity profiles, dividing-streamline positions and induced pressures, as discussed in § 4, is nevertheless encouraging and leads us to conclude that the theory is a good first step towards a fuller understanding of the incompressible blowing problem. Some evidence of separation ahead of the blow at the higher injection rates is also found.

2. The theory

Briefly summarizing, the Cole–Aroesty (1968) theory for strong normal injection, adopted by Wallace & Kemp (1969) and Smith (1972) for subsonic flows and by numerous authors in supersonic and hypersonic problems, assumes a steady laminar motion with, specifically, (*a*) a uniform mainstream, (*b*) a laminar leading-edge boundary layer, (*c*) uniformly distributed laminar injection and (*d*) a steady, laminar and relatively thin blown-off boundary layer separating the main injectant and free-stream inviscid flow regions in $x > 0$ (see figure 1).

Under these assumptions, and taking $S(0) = 0$, we deduce by asymptotic expansion methods that $S(x)$, the dividing streamline position, and $P(x)$, the induced pressure near the plate, must satisfy the following equations for incompressible conditions:

$$(P(x)-P_\infty)/\rho U_\infty^2 L = -\frac{1}{\pi}\int_0^\infty \frac{S'(t)\,dt}{x-t}, \tag{1}$$

$$S(x) = V_w\left(\frac{\rho}{2}\right)^{\frac{1}{2}}\int_0^x \frac{dt}{[P(t)-P(x)]^{\frac{1}{2}}}, \tag{2}$$

the latter being due to Cole & Aroesty (1968) and the former found from incompressible thin-wing theory. Here P_∞ is the free-stream pressure, ρ is the density and we shall use u and v to denote the velocity components in the streamwise

(x) and transverse (y) directions respectively. The plate surface is given by $y = 0$ and blowing commences at $x = 0$. For small values of x/L Smith (1972) has exhibited two solutions to the problem, the first agreeing with Wallace & Kemp (1969):

$$S(x) \sim \delta^{\frac{2}{3}}\{s_0 x + o(x)\}, \tag{3}$$

$$\frac{P(x)-P_\infty}{\rho U_\infty^2} \sim \frac{-\delta^{\frac{2}{3}} s_0}{\pi}\left\{\ln\left(\frac{x}{L}\right) + o\left(\ln\frac{x}{L}\right)\right\}, \tag{4}$$

where $s_0 = (\frac{1}{2}\pi^2)^{\frac{1}{3}}$. The corresponding injection-region profiles have a similarity form

$$u = U_\infty \delta^{\frac{1}{3}}\left(\frac{2s_0}{\pi}\right)^{\frac{1}{2}} \operatorname{erf}^{-1}\left(\frac{y}{S(x)}\right), \tag{5}$$

where erf^{-1} denotes the inverse error function.

The second possibility raised in Smith (1972) is an expansion of the form

$$S(x) \sim \left[a_0\left(\frac{x}{L}\right)^{\frac{1}{2}} + a_1\left(\frac{x}{L}\right)^{\frac{3}{2}} + a_2\left(\frac{x}{L}\right)^{\frac{5}{2}} + O\left(\left(\frac{x}{L}\right)^{\frac{7}{2}}\right)\right]\delta^{\frac{2}{3}}, \tag{6}$$

$$(P(x)-P_\infty)/\rho U_\infty^2 \sim \left[P_0 - P_1\frac{x}{L} - P_2\left(\frac{x}{L}\right)^2 + O\left(\left(\frac{x}{L}\right)^3\right)\right]\delta^{\frac{2}{3}}, \tag{7}$$

and substitution into (1) and (2) shows that the relationships between the coefficients a_i and P_i involve a number of infinite integrals that introduce some degree of freedom into the solution (6) and (7). The injectant velocities here develop according to expansions proceeding in integral powers of x/L.

The comparisons with observed values to be made in § 4 below tend to support the solution (3) and (4) and indeed we shall concern ourselves mainly with this solution in the remainder of the paper, although some comment will be made in § 5 on the relevance of the regular pressure behaviour (7) and the parabolic streamline shape (6). Perhaps more important, however, is the implication in the theory that the velocity profile has an inflexion point and high shear away from the porous surface. Our experiments were designed to 'satisfy' conditions (*a*), (*b*) and (*c*), as was stated in the introduction, but bearing in mind the theoretical suggestions the motion in $x > 0$ could be expected to be unstable and so it proved to be in practice. Condition (*d*), then, could not be satisfied. Any comparisons between the theory and experiment must therefore be made with caution, the former being further deficient in that it also requires $\delta^{\frac{1}{3}} \ll 1$ whereas $\delta^{\frac{1}{3}}$ took values up to 0·375 in the experiments. Despite this, it was intended that quantitative comparisons should be made between the solutions for small x/L and measured values.

3. Experimental arrangements

A sketch of the experimental arrangements is given in figure 1. The experiments were carried out in the low-speed wind tunnel of the Oxford University Engineering Laboratories; the tunnel is capable of speeds from 300 to 3000 cm/s approximately and its working section is 31·75 cm high and 45·70 cm wide. The flat-plate model, which was set vertically between, and screwed to, the top and

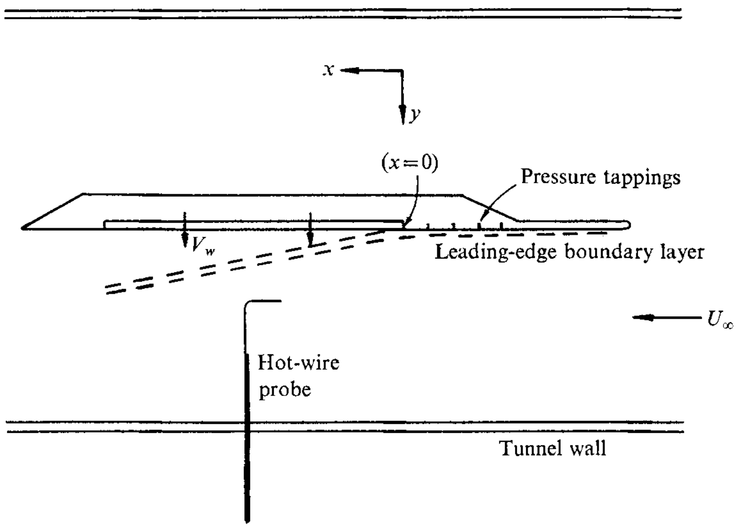

FIGURE 1. Top-view diagram (not to scale) of the flat-plate model. The dashed lines indicate the *theoretical* location and relative thickness of the boundary layer.

bottom tunnel walls, measured 31·75 cm high by 44·47 cm long with maximum thickness $2\frac{1}{2}$ cm and was made of perspex with a porous metal insert 31·75 cm high by 16·47 cm long and 3 mm thick.

Critical to our arrangements was the existence of laminar flow in the mainstream and close to the plate (at least upstream of the blowing region) in the absence of applied pressure gradients, our interest being solely in the effects produced in the flow by the injection. After some initial trials it was found that, with an impermeable test surface that was slightly curved and smoothed to produce a rounded leading edge, the flow showed laminar characteristics in the boundary layer and velocity profiles, measured by hot-wire methods, agreed to within a few per cent with the Blasius theory for a range of U_∞ values. Further details of these measurements are presented by Smith (1972). The zero-pressure-gradient condition imposed here was achieved by adjustment of the angle of the model relative to the tunnel side walls until the readings of 7 static-pressure tappings sunk into the plate at various intervals along its centre-line were equal. The required orientation of the model was in fact within 5° of alignment with the side walls (see Bellhouse (1965) for the effect of the tunnel walls on the pressure gradient).

Then, when the porous unit was included in the test surface, leaving an impermeable leading-edge section of length 21·6 cm upstream with only four pressure tappings retained in it, the laminarity requirements were again satisfied with a uniform static pressure. The unit was prepared from coarse porous stainless-steel sheeting and a plenum was left between it and the back face of the model. The main reason governing this choice of sheet was its rigidity; no honeycomb structure, for instance, which could have caused some blockage of the injection (see McQuaid 1967), was needed to support the sheet and even with our quite strong blowing it remained planar whereas the porous polythene sheets that were tested showed considerable bowing. The porous section was screwed and Araldited into place, ledges $1\frac{1}{4}$ cm wide being used to secure a good screw-fitting at three edges, but not at the porous leading edge. Here, so as not to block the injection, the screws were each fitted into a supporting column behind the sheet and the columns Araldited to the back face; adhesive tape was also used between the solid and porous units. With these arrangements we avoided the bowing of the sheet and fierce air jets which otherwise could occur at the perspex-to-porous joints.

3.1. *The blowing mechanism*

A pressure supply in the laboratory was employed to build up the high pressure inside the plenum required to force air through the porous sheet. The supply was joined to a five-arm manifold outside the working section via a regulator, and the manifold arms then transferred the supply to the plenum chamber through five holes drilled in the top of the chamber. A special venturi was constructed to meter the total flow supplied. Because of the quite large mass flow rate required for the blowing the design had to circumvent the possibility of significant compressibility effects which could arise within venturis of standard design, and so the inner diameters were made 2·29 cm and 2·54 cm. Calibrations with air and water showed, however, very good agreement between the theoretical and measured flow rates within the desired fluid speed range (see also § 3.3 below) and during experimental runs the value of the injection velocity V_w was therefore determined from the readings of the pressure tappings in the venturi.

3.2. *Measurement of velocity profiles and pressures*

To give the required high Reynolds number flows the free-stream speeds U_∞ used were in the range 728–2263 cm/s. The values of U_∞ were measured with a standard Pitot-static tube connected to a multi-channel manometer inclined at 20° for greater sensitivity in its readings. This manometer also took the readings from the four static tappings, situated at $x = -0{\cdot}65, -1{\cdot}35, -3{\cdot}10$ and $-4{\cdot}95$ cm in the perspex leading-edge section, and from the static-pressure probe described below.

Hot-wire anemometers of the constant-temperature type were used to study the flow configuration, the wires being 5 μm in diameter, 7 mm long and of platinum-plated tungsten. The probes and power supply unit, and principle of their operation, are described in Smith (1972), which also contains details of the calibration procedures based on the application of King's (1914) law; the hot wires were all calibrated against the Pitot-static tube in the uniform airflow upstream of the model. Each probe was mounted in a traversing mechanism (with a vernier scale attached to measure y values to within 0·1 mm) and inserted into the tunnel through an opening in the side panel. The x values were determined from a prior calibration on the side panel.

Each experimental run consisted of taking, on average, 20 sets of hot-wire voltage (V_B) readings at 20 different y values for fixed x, U_∞ and V_w, starting at $y = 0$ and moving out in appropriate steps until uniform conditions were attained. An average V_B for each y was then evaluated and converted via the V_B–u calibration curves to give the local fluid velocity u. The $y = 0$ position was ascertained visually by looking down from the top wall and making careful manipulator adjustments to reduce the gap between the wire and wall. The estimated error in this positioning was less than 0·15 mm and this reckoning was substantiated by the very good agreement with Blasius profiles for zero injection. The variation in time of the air velocity at a given position in the flow was also examined. An oscilloscope was connected to the anemometer unit and its trace, following the variations in bridge voltage V_B, could be used as a reliable indicator of any unsteadiness in the flow being probed by the hot wire. In particular, a marked difference could easily be detected between the steady laminar, unsteady laminar and fully turbulent states of flow present at various positions in the tunnel.

A thin static-pressure probe was constructed for measurement of the pressure distribution near the model's surface. The nose was rounded and six tappings were taken 6·35 mm along the probe head, which was 0·71 mm O.D. and 1·14 cm long. The probe's readings were shown to agree with those of the standard probe above when placed in the mainstream and checked in turn with those of the four wall tappings in $x < 0$ for small y distances. The thin probe proved at its most useful in the 'further experiments' described in § 4 and it also showed that for given $x > 0$ the pressure remained constant, within the limits of accuracy of the manometer readings, for distances up to $\frac{1}{2}$ cm from the test surface in a typical experimental run.

3.3. *Uniformity of blow and two-dimensionality*

The variation in the blowing strength was examined for three values of V_w. The tests were carried out in the absence of mainstream flow, it being reasoned that during the experiments the effect on the blowing distribution of pressure variations along the outside of the porous sheet could be neglected when compared with the high pressures inside the plenum. The static pressures clearly must have had some influence on the blowing distribution because the flow rate through the porous unit was governed by the difference in pressure across the unit, but it was found that the plenum pressures, measured throughout the relevant range of blowing speeds, were of such magnitude that the external variation represented at most an 8 % effect on the pressure difference.

The porosity variation was checked with a hot-wire probe normal to the usual upstream-facing direction. Since the blowing distribution consisted in fact of a series of air jets interspersed with solid surfaces, purely local values of V_B recorded at the porous plate were highly dependent upon the position of the measuring probe relative to the air gaps. Accordingly it was necessary to take a local average. About 50 values of V_B were recorded over each of several $2\frac{1}{2}$ cm square areas centred at various positions on the plate, with the hot wire within 1 mm of the surface. The overall variation of V_B average values indicated a variation in V_w of $\pm$ 15 % about the mean value, and the variation decreased rapidly to a few per cent at about 2 mm from the surface. This was considered a reasonable approximation to the ideal uniform distribution, or vice versa. McQuaid (1967), who used a porous sheet supported by a honeycomb arrangement and measured the distribution with a $1\frac{3}{4}$ in. diameter suction-pipe, reported a variation of $\pm 10\%$ in the blow, a figure also obtained by Fernandez & Zukoski (1969) in their experiments.

Furthermore, the agreement between the average values of V_w implied by the venturi readings and those derived from the hot-wire readings supported the contention that King's law gives a good approximation to the hot-wire characteristics even for the small values of velocity measured in our experiments.

The only checks made on two-dimensionality were traverses of the flow from the top to the bottom tunnel wall, using the static-pressure probe, for the two extreme blows given in § 4. The probe was kept at a fixed x value and a small y distance. For both the injection rates the spanwise variation in pressure was satisfactorily small across most of the plate, and indeed the pressure deviated significantly from the mean mid-plate values only very near the tunnel walls, where, because of the $1\frac{1}{4}$ cm supporting ledges, there was no injection and the air speed was therefore close to the mainstream values.

4. Experimental results and comparison with theory

Velocity profiles were measured for various δ and mainly at stations $\frac{1}{2}$, 1, 2, 3, 4 and 8 cm downstream of the start of the blow. Figure 2 shows samples of our results, δ being kept approximately constant in each graph and x being varied; a number of results obtained at intermediate stations are also included, along with the profiles upstream of injection for each blow. Taking the Reynolds number based on the leading-edge length $L = 21{\cdot}65$ cm, the values of, for instance, $\delta/Re^{-\frac{1}{2}}$, which the theory assumes to be $\gg 1$, vary from about 4 for $\delta \approx 0{\cdot}008$ to about 10 for $\delta \approx 0{\cdot}053$. Most profiles downstream of $x = 0$ exhibit a blown-off boundary-layer form in which the zone of highest shear occurs at a distance from the plate and the u profile has an inflexion point. Apart from results for $\delta \approx 0{\cdot}012$, where the Reynolds numbers varied appreciably, all the cases examined followed a pattern in which a steady increase in wall-layer thickness resulted from an increase either in x for fixed δ or, comparing the separate graphs, in δ for fixed x.

As regards experimental errors, our measured profiles in $x > 0$ are bound to be suspect very close to $y = 0$ for a number of reasons, including the following: (i) hot wires are not directional and measure the cooling effects of the total velocity rather than just u; although the streamwise velocity was indeed dominant over the majority of the flow, ideally $u = 0$ and $v = V_w$ at $y = 0$, suggesting that values of u will be overestimated; (ii) wire characteristics almost certainly vary with the direction of the oncoming flow; (iii) there were possibly thermal effects due to the close proximity of a conducting metal surface; (iv) King's law becomes less appropriate for the smaller velocities near $y = 0$. No results are presented for readings at $y = 0$. For all blows the values of u/U_∞ there were usually between 0·05 and 0·15 and were greater than δ; it is felt that this was chiefly because of effects (ii) and (iii). We emphasize that effects (i)–(iv) apply only within approximately 0·2 mm of the plate and that the measured values for u away from the plate should be very accurate. Only the following errors are likely to have had any significant influence on all the wall-layer measurements: (v) the

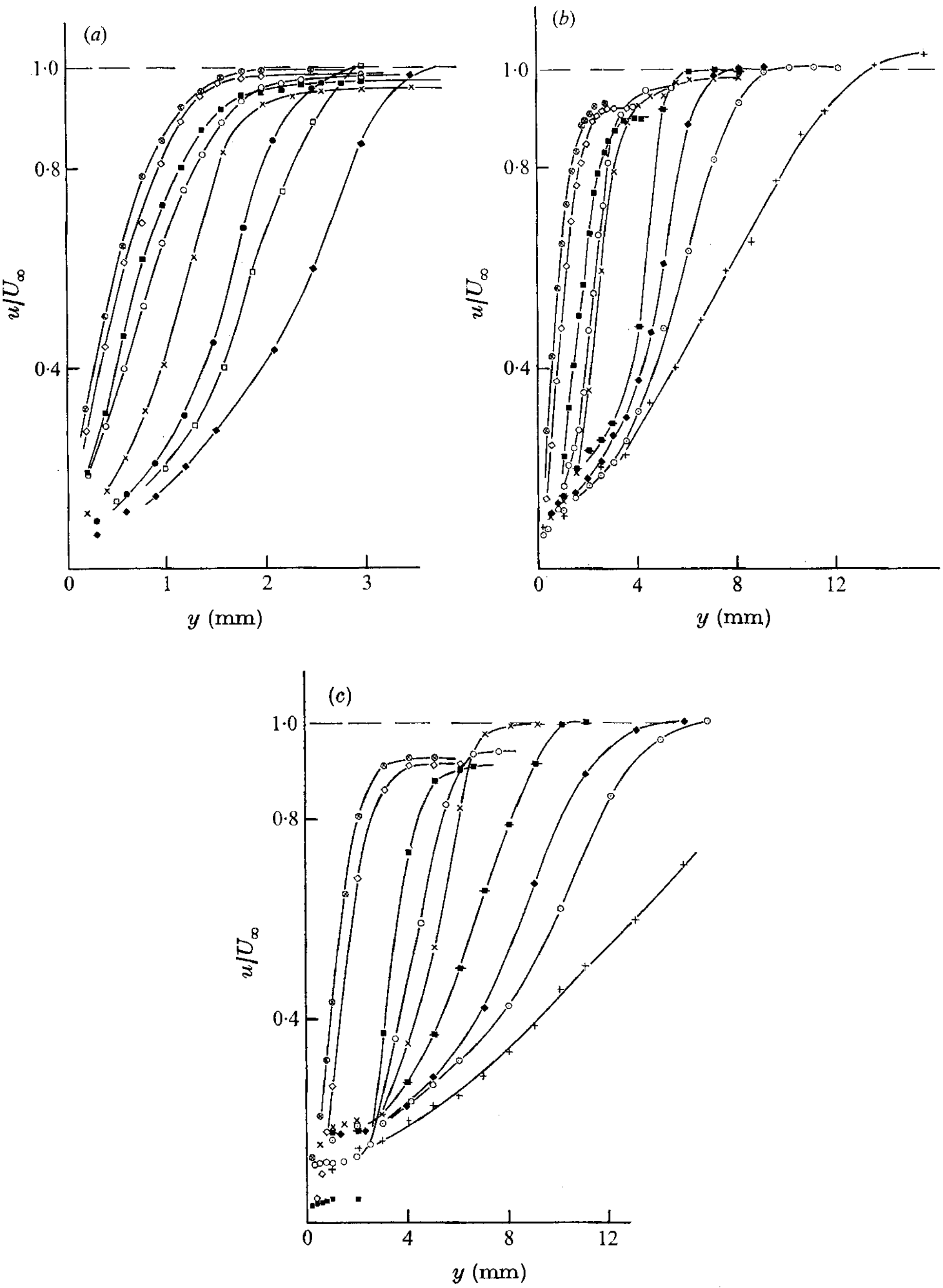

FIGURE 2. Measured velocity profiles. (*a*) $\delta = 0{\cdot}0086 \pm 0{\cdot}0001$, $U_\infty = 1587 \pm 2$ cm/s (except for $x = 1{\cdot}5$ and $2{\cdot}5$ cm, where $\delta = 0{\cdot}0078 \pm 0{\cdot}0002$, $U_\infty = 2235 \pm 33$ cm/s, and $x = 3{\cdot}0$ cm, where $U_\infty = 2210$ cm/s). (*b*) $\delta = 0{\cdot}0242 \pm 0{\cdot}0002$, $U_\infty = 1054 \pm 4$ cm/s. (*c*) $\delta = 0{\cdot}0527 \pm 0{\cdot}0010$, $U_\infty = 732 \pm 4$ cm/s.

	⊗	◇	■	○	×	●
x (cm)	−4·95	−3·10	−0·65	0·5	1·0	1·5
	-■-	□	◆	⊙	+	
x (cm)	2·0	2·5	3·0	4·0	8·0	

zero-position error discussed in § 3; (vi) any oscillations of the wire produced by forces exerted by the airflow or by vibration of the traversing gear. This last would have had greatest effect in the regions of high shear. Also, it must be noted that the blow was very non-uniform when purely local values of V_w were considered although the overall variation in the blowing probably had little overall effect (cf. McQuaid 1967).

As expected the motion of the fluid in the high-shear regions was not laminar but exhibited unstable laminar characteristics. The nature of the flow was established from the voltage traces on the oscilloscope, which, with the hot wire placed in the shear layer, showed a motion neither fully laminar nor fully turbulent. The former was characterized by a relatively steady trace whereas the latter type of flow produced an irregular, spiky trace, the voltmeter readings showing respectively small and large amounts of scatter. (Examples of typical traces for turbulent and laminar flows are given by Bellhouse (1965) and Brown (1967).) The traces obtained for the shear-layer flow were however of an intermediate type which might be described as a regular oscillatory laminar form, with only very occasional small bursts of turbulence. Moreover, the division of the $x > 0$ flow field into three distinct regions, as envisaged in § 2, could be observed by referring to the oscilloscope trace as the hot wire was moved away from the porous surface. Particularly in the cases of strong injection, in which viscous effects were truly blown off, laminar flow prevailed near the wall; this was followed by an unsteady (shear) layer; finally the fluctuations died down as the mainstream was approached. For weaker blows the distinction remained, but to a lesser extent because of the thickening of the shear layer and its effects due to unsteadiness. On the other hand the scatter of the shear-region traces for stronger blows was greater than that in a weaker case, and the shear-layer unsteadiness also increased as the hot wire was moved downstream.

From these observations the applicability of the theory developed in § 2 may be assessed qualitatively. First, there are indeed three basic regions as was assumed theoretically, but in our experiments we found the differences between the regions less marked in some cases. Second, the theory has an obvious deficiency in that it assumes a thin dividing layer whereas in practice the layers were always unsteady and, as shown in figure 2, could not always be regarded as being relatively thin. Consequently, although the theory is reasonable in some respects, the parts played in the formation of the shear layer both by mixing of the two fluid supplies and, more especially, by instability in the shear layer must be emphasized.

To guage the theoretical model quantitatively we compare the measured results with the predictions of § 2. An estimate for the value of $S(x)$ was made using all the measured velocity profiles. The procedure assumes no mixing of the injectant and free-stream air supplies and is based simply on the mass conservation principle:

$$\frac{V_w x}{U_\infty} = \int_0^{S(x)} \frac{u(x,y)}{U_\infty}\,dy. \tag{8}$$

For given x, δ and velocity profile, $S(x)$ can thus be evaluated iteratively by integration of the velocity curves in figure 2. For this purpose the profiles were

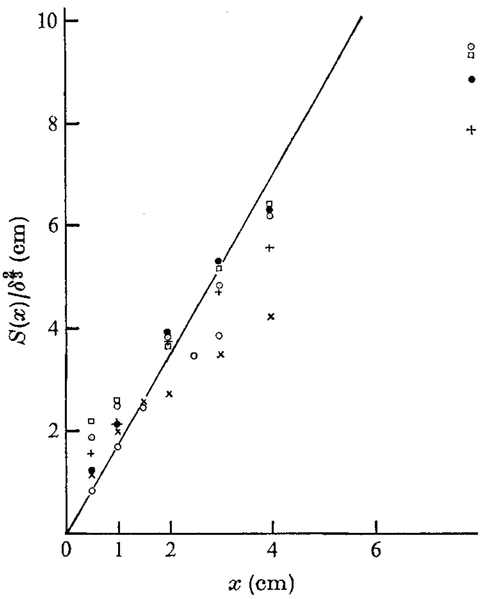

Figure 3. The position $S(x)$ of the dividing streamline. ——, theory.

	○	×	●	+	ↄ	□
δ	0·008	0·012	0·0176	0·024	0·036	0·053

extrapolated to $u = 0$ at $y = 0$, and it is noteworthy that the influence of the suspect results near $y = 0$ on the values of the integrals was negligible. The theory is now tested by expressing all the results in the form $S(x)/\delta^{\frac{2}{3}}$ *vs.* x as in figure 3 and we see that the theoretical shape (3) proposed for a small range of x values only is in reasonably good agreement with the form derived from measured velocity profiles for the smallest blow $\delta \approx 0{\cdot}008$ between $x = 0$ and 2 cm, and is not inconsistent for the larger blows for the whole range $0 \leqslant x \leqslant 3\frac{1}{2}$ cm *apart from near* $x = 0$. At $x = \frac{1}{2}$ and 1 cm the estimated values for the strongest blow are as much as $2\frac{1}{2}$ times the theoretical values. The overall shape of the estimated $S(x)$ is convex upwards and in all cases it eventually moves away from the theoretical small-x curve.

Figures 4 (*a*)–(*c*) show further comparisons. The respective velocity profiles in $x > 0$ are plotted as $u/U_\infty \delta^{\frac{1}{3}}$ against $y/S(x)$ for $0 \leqslant y/S(x) \leqslant 1$ (the injectant region), where $S(x)$ has been evaluated above. Here again the theory (equation (5)) is broadly in line with the measured values and, as in the previous paragraph, the agreement generally improves as the blow rate decreases, although the difficulties at or near $y = 0$ seem to pull all the measured values away from the theoretical curve. We remark that comparison here corresponds to roughly one third only of the total profile for a typical traverse. Further the theory, taken to first order only, predicts a singularity in the velocity as the dividing streamline is approached and, more significantly perhaps, the value of $S(x)$ used to normalize y in figure 4 is approximately twice the theoretical value given by (3) for some stronger blows.

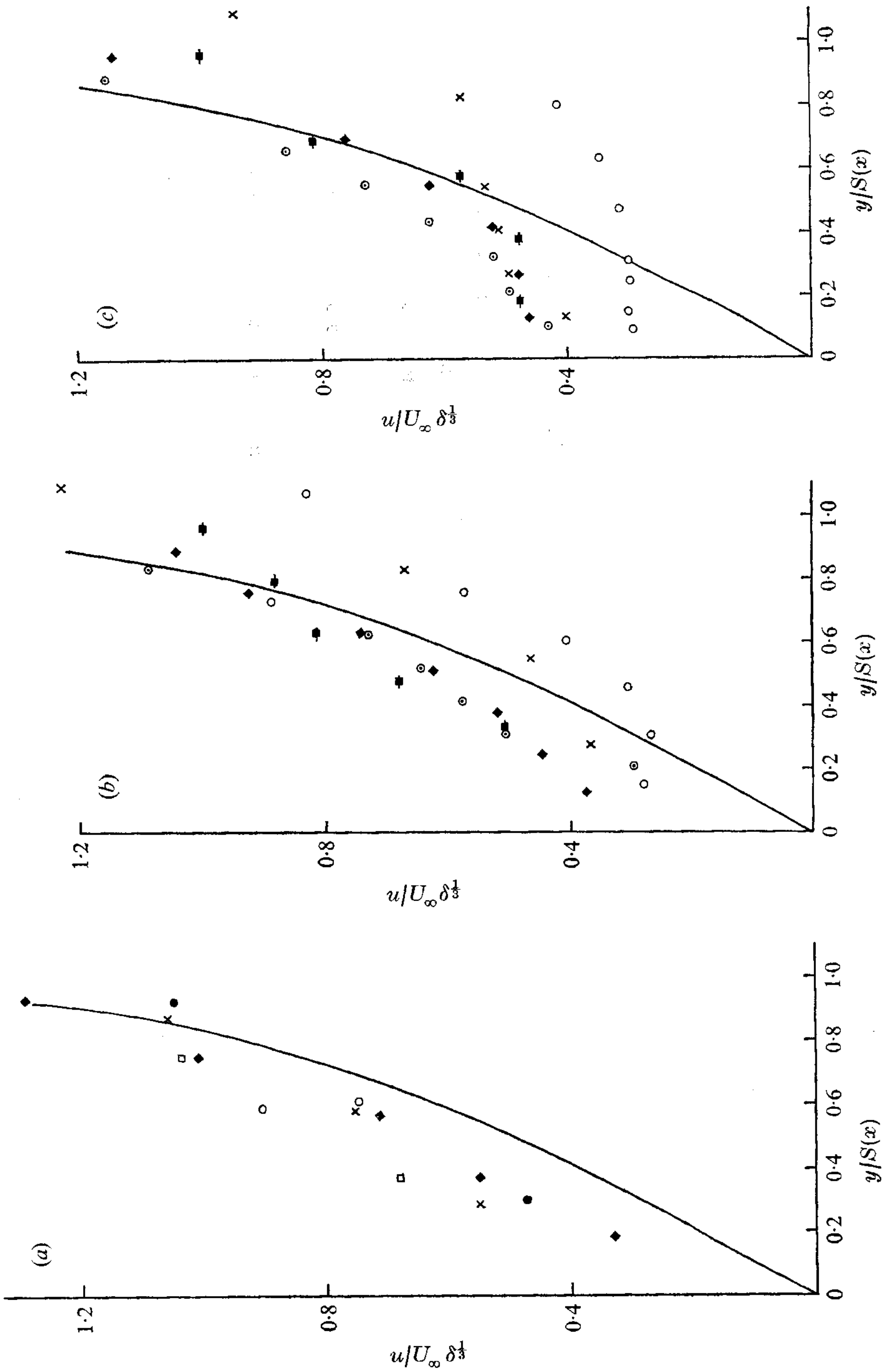

FIGURE 4. Injectant-region profiles for (*a*) $\delta \approx 0{\cdot}008$, (*b*) $\delta \approx 0{\cdot}0242$, (*c*) $\delta \approx 0{\cdot}0527$. Symbols and exact values are as in figure 2. ——, theory.

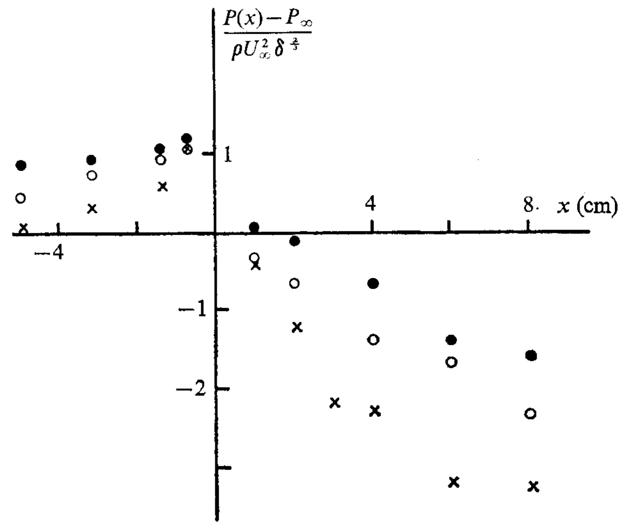

FIGURE 5. Non-dimensionalized pressure distributions. ×, $\delta = 0{\cdot}0086$, $U_\infty = 1596$ cm/s; ○, $\delta = 0{\cdot}0289$, $U_\infty = 974$ cm/s; ●, $\delta = 0{\cdot}0528$, $U_\infty = 732$ cm/s.

The distribution of static pressure $P(x)$ along the length of the model is shown in figure 5 for three blows. All the pressures in $x > 0$ were taken at a small distance from the test surface, after first checking the variation with distance y to ensure that the constant value in the wall layer had been reached, while the results for $x < 0$ were calculated from the readings of the wall tappings. In figure 5 the pressure perturbation $P(x) - P_\infty$ is scaled with a factor $\delta^{\frac{2}{3}}\rho U_\infty^2$ (stemming from (4)). Although the total pressure rise ahead of the blow now seems to be constant for all the δ's, the graphs for each δ are quite distinct both in $x < 0$ and in $x > 0$. Equation (4) predicts a universal curve for this graph for small x/L and the differences away from $x = 0$ may be attributed to the upstream influence of both the start and the finish ot the blow. In fact, if the scaled pressures for $x > 0$ are plotted against $\ln x$ (since (4) implies a straight line for some small x/L) the agreement between the theoretical and experimental slopes is found to be reasonable.

Concerning assumption (*b*) in § 2, laminar conditions were found to prevail in the leading-edge boundary layer for non-zero injection in the range of injection rates that were studied. Shown with the $x > 0$ profiles in figure 2 are the impermeable-region profiles for $x = -0{\cdot}65$, $-3{\cdot}10$ and $-4{\cdot}95$ cm, stations coinciding with three of the wall pressure tappings. The leading-edge boundary-layer profiles are given separately in figure 6 so that comparison between the upstream influences of the three blows may be made. For the two lowest blow rates the profiles compare well with the theoretical Blasius profile at the farthest upstream station; on the other hand, the thickening (compared with the Blasius no-blow values) of the boundary layer for $\delta = 0{\cdot}0528$ is quite dramatic, even at $x = -4{\cdot}95$ cm. The trend of the profiles for fixed blowing is as in the $x > 0$ region: the layer thickness increases with distance downstream. At $x = -0{\cdot}65$ cm, moreover, for $\delta = 0{\cdot}0528$ the shear remains small and approximately constant up to

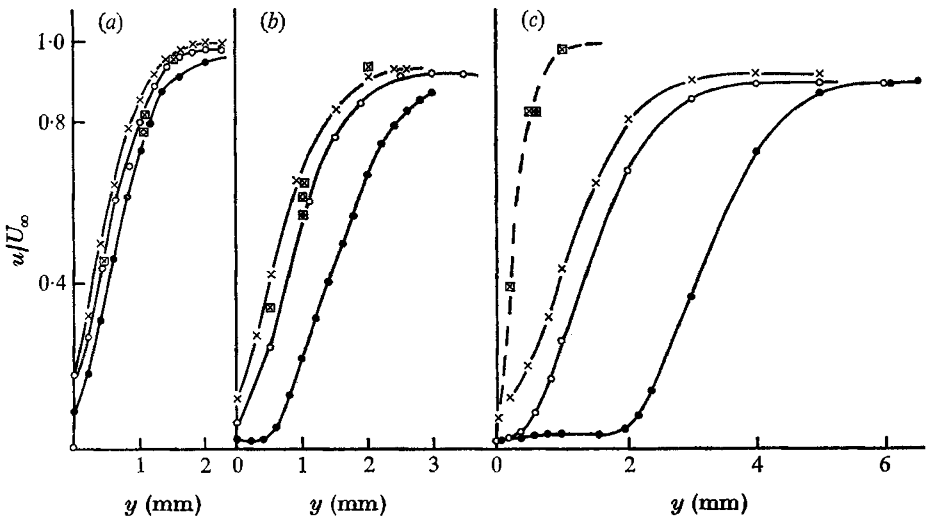

FIGURE 6. Velocity profiles upstream of injection. (a) $\delta = 0{\cdot}0086$, (b) $\delta = 0{\cdot}0244$, (c) $\delta = 0{\cdot}0528$. ×, $x = -4{\cdot}95$ cm; ○, $x = -3{\cdot}10$ cm; ●, $x = -0{\cdot}65$ cm; boxed symbols indicate corresponding Blasius values.

2 mm from the wall before rising sharply and a similar phenomenon is apparent in the $\delta = 0{\cdot}0244$ profile at $x = -0{\cdot}65$ cm. Because the hot-wire voltages depend only on the magnitude of the probed flow and no account is taken of direction, this could well indicate a separation in $x < 0$ and a zone of reversed flow upstream of the injection, no evidence of any separation within the blown region being found. In supersonic flows, Fernandez & Zukoski (1969) have also reported upstream separation occurring at their higher strong blowing rates and some signs of it can be found in Hartunian & Spencer's (1967) studies of massive blowing. In our experiments the traces on the oscilloscope were always of the laminar type in $x < 0$ but tended more towards the unstable sort, typical of the shear layer in $x > 0$, near the porous leading edge, especially for the strongest blowing rate.

Finally in this section we note that a further series of related experiments was conducted by the author (1972) in which a thin 100-gauge Mylar sheet, secured to the plate at $x = 0$ but free to move in $x > 0$, was used to divide the injected air from the mainstream supply. One purpose was to gauge the effect of mixing in the viscous shear layer, and measurements of pressures and velocity profiles within the injectant region and of the average positions ($S(x)$) of the sheet were taken. These $S(x)$ values agreed to a slightly higher degree of accuracy with first-order theory, equation (3), than did those in the original set-up and the agreement in pressures and velocity profiles was comparable with that in figures 2 and 5 although the scatter in the hot-wire readings for each u measurement was now greatly amplified.

5. Concluding remarks

(i) The shear-layer zone joining the two air supplies was found experimentally to be unstable even though the mainstream, leading-edge boundary layer and injection supply were all predominantly laminar and steady. On the other hand, the shear layer was certainly not turbulent; the flow in $x > 0$ did divide into three regions somewhat similar to those in figure 1, although the distinction between the regions was less marked, particularly for weaker blows; and in general the agreement between the theory and the measured results was encouraging (see (iv) below).

(ii) In the light of (i) we conclude that the steady laminar theory of § 2 must be regarded only as a first step towards a complete understanding of the laminar incompressible blowing problem, but as such it stands fairly well in comparison with the practical situation. Apart from the shortcoming of the theory implied in (i) there were of course other factors that limited the agreement between observation and theory. These included any errors in our measurements and the fact that the theory is taken to first order only and is strictly valid only in an asymptotic sense whereas experimental values of $\delta^{\frac{1}{3}}$, for instance, were up to 0·375.

(iii) Overall the experimental findings favour the linear form (3) for the initial growth of the dividing streamline. A comparison between the results and the $S \sim a_0 x^{\frac{1}{2}}$ solutions (equation (6)) has not been attempted even though some agreement may be possible, making use of the arbitrariness in the a_i and P_i of (6) and (7). It could well be in fact that the regular pressure behaviour (7) near $x = 0$ is more appropriate to the situation envisaged in (iv) below where $S(0) > 0$ and the boundary layer is blown off upstream of the injection, a phenomenon suggested to some extent by the normal direction of the dividing streamline in (6) at $x = 0$. In the Mylar sheet experiments, where mixing effects were negligible, the straightness of the dividing streamline was preserved somewhat further downstream for all δ, and, ignoring the influence of the barrier on the flow, the large degree of instability in the flow could be observed in the scatter ot the hot-wire readings. This would seem to indicate that, in the formation of the shear layer, mixing effects, although not small, were not as important as the inherent instability present.

(iv) The most likely reason for the rather high initial values of the injectant-region thickness, as implied by the high values of $S(x)$ and compared with the theoretical predictions, when the blow was increased is that separation took place ahead of the porous plate. The leading-edge boundary-layer profiles for $\delta = 0{\cdot}0528$ showed signs of a possible reversed flow at $x = 0{\cdot}65$ cm, as well as a large increase in the leading-edge boundary-layer thickness even 4·95 cm upstream of $x = 0$. The accompanying adverse pressure gradient also developed, in anticipation of the strong blow at $x = 0$, at a large distance upstream (figure 5) whereas the $\delta = 0{\cdot}0086$ leading-edge boundary-layer profiles and pressure distribution changed significantly from their no-blow values only well within a distance 0·65 cm of the blow. The onset of unstable and perhaps turbulent effects that presumably accompanied the separation could explain the increase in scatter of the V_B readings in the shear layer for stronger blows, and the genuine blow-off of the boundary layer would give rise to the marked difference between wall-layer, shear-layer and mainstream regions that was observed even at $x = \frac{1}{2}$ cm. The results in § 4 point to a crossover from the $S(0) = 0$ form assumed in § 2 to an $S(0) > 0$ form (separation ahead of the blow) between $\delta = 0{\cdot}024$ and $\delta = 0{\cdot}053$, and the possibility is thereby raised of a subsonic analogy of the supersonic theory developed by Smith & Stewartson (1973) for strong distributed injection into a separated boundary layer.

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